

Spherical aberration experimental apparatus for undergraduate optics courses

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ABSTRACT: The understanding of the phenomena of aberration of lenses and mirrors is important for undergraduate students in university courses related to optics. However, most educational experiments usually are limited to the study of lenses and mirrors of small aperture, where spherical aberration is considered negligible. A simple experimental apparatus was developed, which allows students to carry out experiments using a non-ideal convex mirror with large aperture. Based on the simplicity of the experimental apparatus and the straightforward measurement procedure, students are able to take multiple measurements of the mirror focal length and observe the variation of the focal length at different distances from the optical centre of the mirror. There is evidence that the experiment motivates students to study related issues and can significantly help the educational process.

INTRODUCTION

In recent years, there has been renewed interest in developing new educational methodologies and experiments for the introduction of geometrical optics in middle school [1-3] and the explanation of more complex issues in undergraduate courses [4-6]. The optical phenomena related to geometrical optics have been adequately studied and widely included in undergraduate courses in optics. Aberrations of lenses and mirrors are considered in undergraduate programmes in optics, optical instrumentation, light detection and surveying. However, explaining and experimentally demonstrating the phenomena has been difficult, since there are few experimental apparatuses that can be used in a student laboratory. Most experiments in student optics laboratories are limited to the study of lenses and mirrors of small lens aperture, where spherical aberration is considered negligible.

Proposed in this article is a simple experimental apparatus to measure the spherical aberrations of a non-ideal convex mirror with large aperture. For this purpose, an optical bench has been made to control and accurately position the optical components for the experiment. The focal length, as well as the radius of curvature of a non-ideal convex mirror, can be measured experimentally and explained in the laboratory, because of the simplicity of the experimental apparatus and the straightforward measurement procedure. The simplified theory regarding spherical mirrors, their properties and spherical aberration can be introduced to the students before starting the experiment.

SIMPLIFIED THEORY

A mirror is a smooth and polished surface that reflects the light incident on it. Spherical mirrors have a reflective spherical surface, while aspherical mirrors have a curved reflective surface which deviates from a sphere, such as hyperbolic, elliptical or parabolic [7-9]. Spherical mirrors are easier and cheaper to fabricate than aspherical mirrors. Spherical aberrations are defects caused by the surface of mirrors which result in different focal length for the light rays parallel to the optical axis (paraxial rays) as compared to those far from the optical axis (non-paraxial rays). In this work, a simplified theory of the spherical aberrations of a convex mirror is presented for students to understand the concepts and be able to calculate the mirror focal length and radius of curvature using the proposed experimental setup.

Ideal Spherical Mirror

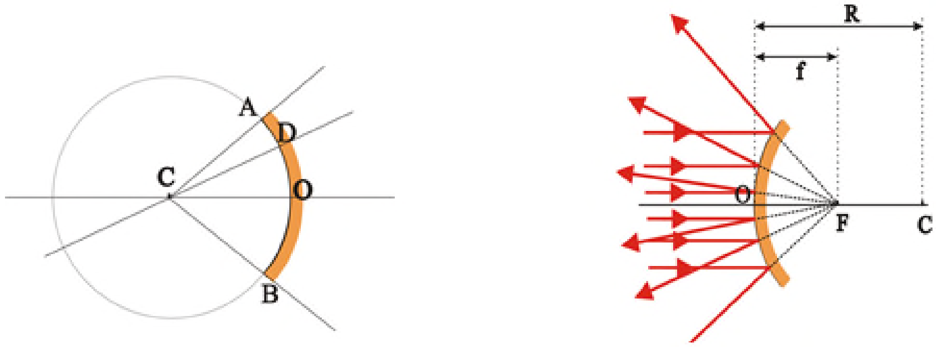
The main characteristics of a spherical mirror are shown in Figure 1a, and are summarised thus:

a) Centre O of the reflective surface (pole of the spherical mirror).
b) Centre of curvature C of the spherical surface of the mirror.
c) Radius of curvature CO of the spherical surface.

d) Straight line through C and O is the optical (or principal) axis of the mirror.

e) Every line passing through the centre of curvature of the mirror C and from a point D of the reflective surface of the mirror other than O is a secondary axis of the mirror.

f) Angle $A\hat{C}B$ defined by the centre C of the spherical mirror and the end points A and B of the reflective surface is the aperture of the mirror. When the mirror has small aperture, it is considered ideal.



a) b)

Figure 1: a) spherical mirror: CO is the optical axis, CD is a secondary axis; and b) the focal point F of an ideal convex mirror, which is defined by the reflection of parallel to the principal axis rays.

Ideal spherical mirrors having the reflective material on the external surface of the spherical shell are called *ideal convex spherical mirrors*. Light rays incident on the convex mirror parallel to the optical axis (Figure 1b) will be reflected diverging as if they originate from a common virtual point F, the focal point or focus of the mirror. For an ideal mirror with small aperture the distance FC is approximately:

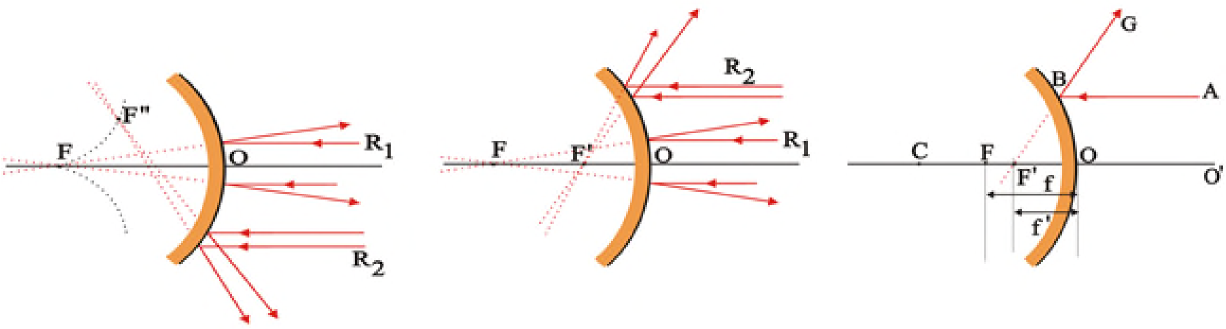
$$FC = FO = \frac{R}{2} \tag{1}$$

The distance FC between the focal point F and the pole O of the mirror is the focal length of the mirror.

$$FO = f = \frac{R}{2} \tag{2}$$

Spherical Aberration of a Convex Mirror

The previous analysis of spherical mirrors was based on the assumption that the mirror has small aperture, therefore the incident ray is exceedingly close to the optical axis, making it an ideal mirror. If the aperture is big enough, rays parallel to the optical axis of the mirror do not diverge from the same point, *the focal point*, but they diverge from points F", positioned at the circumference of a circle (Figure 2a). In this case, the virtual image of a point source is not a single point but a spot with non-infinitesimal dimensions and the image will be blurred (spherical aberration of a convex mirror).



a) b) c)

Figure 2: a) divergence of paraxial rays R_1 and non-paraxial rays R_2 incident on a convex mirror; b) the extension of the reflected ray AB (non-paraxial) intercepts the optical axis of the mirror at F′ instead of the focus F; and c) foci of the paraxial (focal length f) and non-paraxial rays R_1 and R_2 (focal length f′).

Rays R_1 parallel and close to the optical axis of the mirror (paraxial) and rays R_2 (non-paraxial) at a greater distance from the optical axis appear to have their foci F and F" at different points. It is a good approximation to consider that all the foci of the non-paraxial rays R_2 are placed at the point F' of the optical axis of the mirror as shown in Figure 2b.

To calculate the focal length of the convex mirror, a ray AB parallel to the optical axis and at a distance d from it is used (Figure 2c). After reflection the extension of the reflected ray will intercept the optical axis at the point F', which is a distance f′′ from the apex of the mirror.

The relation between f′, the radius of curvature R and the distance d by the geometry presented in Figure 3 is:

$$\frac{f'}{R} = 1 - \frac{1}{2\sqrt{1-\left(\frac{d}{R}\right)^2}} \tag{3}$$

To derive Equation 3, the reflected ray BG due to the incidence of the non-paraxial ray AB is used as shown in Figure 3.

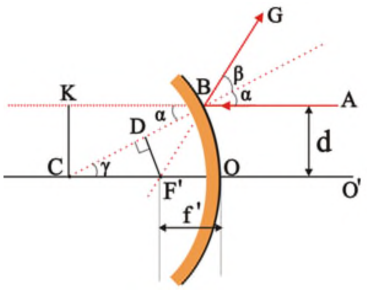


Figure 3: Schematic representation for the calculation of the spherical aberration of a convex mirror.

F′ is the point where the extension of the reflected ray BG intercepts the optical axis and by applying the second law of reflection:

$$\alpha = \beta \tag{4}$$

From Figure 3 since AK and OC are parallel:

$$\alpha = \gamma \tag{5}$$

The triangle F′BC is isosceles and F′D is both median and height, thus DC = R/2 and for the right-angled triangle F′DC, it can be derived that:

$$\cos\gamma = \frac{DC}{F'C} \rightarrow F'C = \frac{R}{2\cos\alpha} \rightarrow$$

$$F'C = \frac{R}{2\sqrt{1-\sin^2\alpha}} \tag{6}$$

From the right-angled triangle CKB:

$$\sin\alpha = \frac{d}{R} \tag{7}$$

By combining the last two equations $OF' = OC - F'C \rightarrow f' = R - F'C$, which finally leads to:

$$\frac{f'}{R} = 1 - \frac{1}{2\sqrt{1-\frac{d^2}{R^2}}} \tag{8}$$

In the case of increasing radius of curvature or for rays close to the optical axis of the mirror $d/R \rightarrow 0$ and Equation 8 leads to $f' = R/2$ as expected for an ideal mirror (Equation 2).

EXPERIMENTAL SETUP AND RESULTS

The experimental setup for measuring the focal length of a convex mirror is schematically shown in Figure 4 and a photograph of the apparatus is shown in Figure 5.

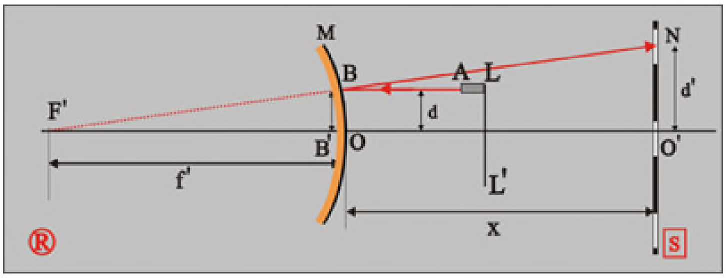
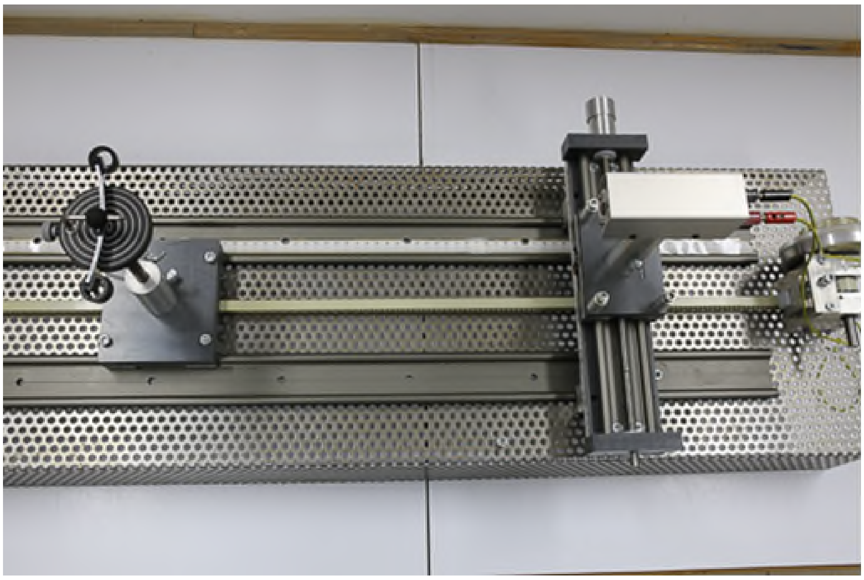
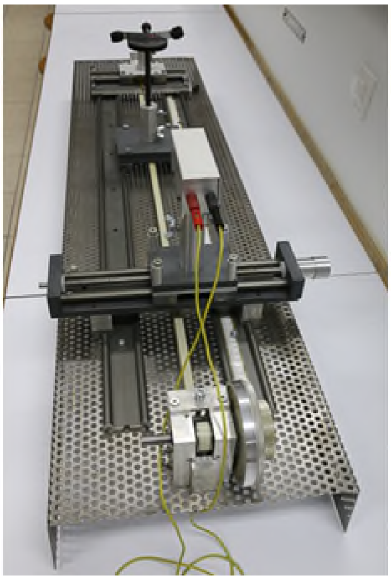


Figure 4: Schematic representation of the experimental setup.

All optical components are placed on a bench R. More specifically, the convex mirror M is placed on its mount B, which is connected to the bench, so that the mirror can be moved parallel to the optical axis OO'. A laser L is the light source, with the beam AB parallel to the optical axis of the mirror. The laser is placed on a mount that is movable in a direction perpendicular to the optic axis of the mirror. A scale LL´ is placed under the laser source and a screen S behind the laser source is applied to measuring the spot of the reflected beam.



a)



b)

Figure 5: Photographs of the experimental setup; the laser beam is reflected by the convex mirror (shown at the left of the photo in Figure 5a and at top in Figure 5b).

Initially the laser source is aligned exactly on the optical axis of the mirror. By moving the laser perpendicular to the optical axis by a distance d, the reflected spot is moved from O´ to N (distance d´). From the similar triangles F´OB and F´O´N the focal length of the mirror for rays displaced by d from the optical axis of the mirror:

$$\frac{F'O}{F'O+OO'}=\frac{BB'}{NO'}\rightarrow\frac{f'}{f'+x}=\frac{d}{d'}\rightarrow$$

$$f'=\frac{dx}{d'-d} \quad (9)$$

The radius of curvature R of the mirror can be derived (to first order approximation) by solving Equation (3):

$$R=f'+\sqrt{f'^2+\frac{d^2}{2}} \quad (10)$$

In Figure 6, the focal length normalised by the radius of curvature of the mirror R is shown with respect to the normalised distance. The solid line is the theoretical curve derived from Equation 3 and the discrete points are the experimental measurements for various values of d. The focal length f´ is negative because the mirror is convex. The agreement between the experimental results and the theoretical curve is excellent. It is not possible to record measurements for small distances d because, in this case, the laser source acts as an obstacle for the reflected laser beam.

The experimental data were collected by students working in groups of two to move the laser and simultaneously measure the distance of the reflected spot from the mirror's principal axis. The experimental procedure was easy to understand by the students and the experimental data were collected in a time period of 30 minutes after the presentation of the simplified underlying theory. It was verified that the experimental setup provides a reliable and accurate method for students to examine and understand the spherical aberration of convex mirrors (i.e. deviation of focal length f΄ from that of an ideal mirror). The focal length f΄ and the radius of curvature R of a non-ideal convex mirror with large aperture were measured accurately in a straightforward way.

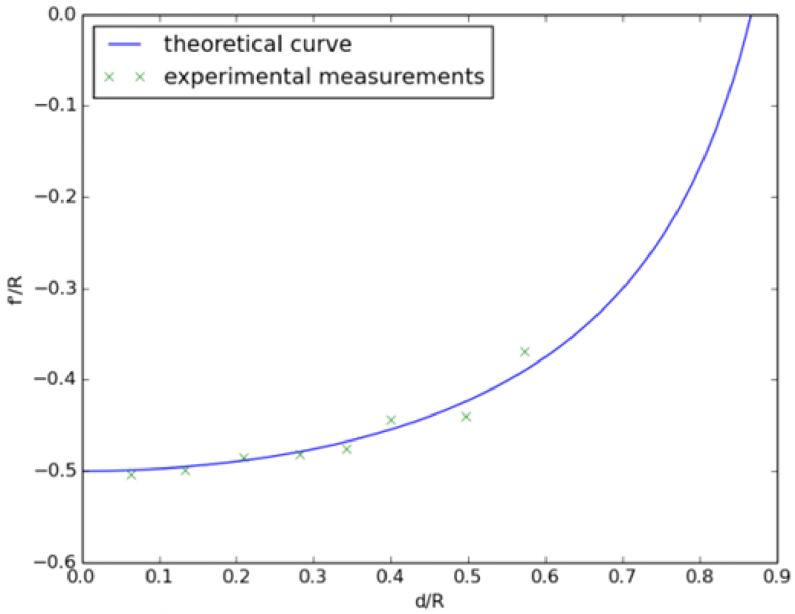


Figure 6: Plot of f΄/R versus d/R for a convex mirror. The solid line is the theoretical curve and the discrete points are the experimental measurements.

In addition, it was observed that when the ratio d/R increases, the divergence between theoretical and experimental rates increases as well. This can be understood by the approximations made in the simplified theory presented, but it can also be used as a starting point to present a more detailed theory of the spherical aberration suitable for students whose specialisation is optics.

CONCLUSIONS

A simple experimental method is proposed that allows students to carry out experiments with a non-ideal convex mirror having large aperture. The focal length of the non-ideal convex mirror can be calculated by the students manipulating the developed experimental apparatus. The deviation of the focal length from that of an ideal mirror and its dependence on the distance from the optical axis of the mirror can be understood by the students and related to spherical aberration theory. Based on the simplicity of the experiment and the straightforward measurement procedure, the proposed experimental apparatus can significantly help the educational process, because it can motivate students to study more complex issues of geometrical optics.

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